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Compressibility and crystal structure of andalusite at high pressure

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Abstract

The unit-cell dimensions and crystal structure of andalusite Al_2SiO_5 have been refined from X-ray data on single crystals mounted in a diamond anvil cell at pressures of 12, 25, and 37 kbar. Structure refinements with anisotropic temperature factors yielded weighted R factors of 3.4, 4.9, and 5.2% respectively. The bulk modulus of andalusite is 1.35±0.10 mbar and the axial compression ratios of orthorhombic unit-cell axes $a:b:c$ are approximately 2.1:1.5:1.0. The relatively greater compressibility of the Al(1)–O_D bond results in a being the most compressible axis. Those bonds that compress $>3\sigma$ between 1 bar and 37 kbar at room temperature, are the bonds that also expand significantly between 25 and 1000°C at room pressure. Polyhedral bulk moduli for the Al(1) octahedron, the Al(2) trigonal bipyramid and the Si tetrahedron are 1.3±0.2, 1.6±0.5, and 4.1±1.5 mbar, respectively. Thus, the aluminum polyhedra are significantly more compressible than the silicon tetrahedron. The omega step-scanning technique of X-ray intensity data collection results in a significant improvement in accuracy and is recommended for structure determination with the diamond-anvil high-pressure cell.

Introduction

High pressure structure determination contain valuable data on the equations of state, interatomic forces and chemical bonding in minerals. The Al_2SiO_5 polymorphs, andalusite, sillimanite and kyanite, provide an interesting system where aluminum occurs in three types of coordinations; in addition to the octahedral coordination found in all three minerals, aluminum also occurs in tetrahedal coordination in sillimanite and in five-fold trigonal bipyramidal coordination in andalusite. Knowledge of the response to pressure and temperature of these different types of Al–O bonds in the presence of relatively rigid SiO_4 tetrahedra is necessary for an understanding of the stability relations and phase transformation within the Al_2SiO_5 system. The temperature effects on the crystal structures of andalusite, sillimanite and kyanite have been determined by Winter and Ghose (1979). The elastic constants of andalusite and sillimanite have been determined by Vaughan and Weidner (1978), who used the Brillouin scattering technique. From infrared and Raman spectroscopic data, the phonon spectra of andalusite and their temperature dependence have been determined and interpreted on the basis of a rigid-ion model by Iishi et al. (1979).

In this paper are presented data on the high pressure structural response of andalusite to 37 kbar, as well as correlations between high-pressure changes and high-temperature response, elasticity, and phonon spectra. It has been possible to observe directly the structural elements of andalusite that are responsible for the compressibility and, hence, the elastic constants. The present data on andalusite at high-pressure, combined with high-temperature data of Winter and Ghose (1979), may be used to test the inverse relationship between structural responses due to temperature versus pressure (Hazen and Finger, 1982). Andalusite is a light-atom structure with a small unit-cell and orthorhombic symmetry and thus also constitutes a test case for the improvement in accuracy of X-ray intensity data collected at high pressure by the omega step-scanning technique. The resulting improvement in

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Table 4a. Calculated and observed structure factors for andalusite at 12 kbar; continuous scan data.

H	OBS	CALC	H	OBS	CALC	H	OBS	CALC	H	OBS	CALC	H	OBS	CALC	H	OBS	CALC	H	OBS	CALC
0 K 0			**8 K 0**			**5 K 1**			**2 K 2**			**0 K 3**			**8 K 3**			**8 K 4**		
0	0	3200	0	569	585	9	286	276	8	184	194	5	148	150	6	69	78	0	430	439
2	163	165	1	152	147				9	312	313	7	194	208				1	157	111
4	488	480	2	72	40	**6 K 1**			10	63	55	9	105	85	**9 K 3**			2	69	41
6	216	202	3	69	65							11	68	79				4	142	147
			4	263	191	1	83	41	**3 K 2**						0	141	123	5	80	20
1 K 0			5	65	27	2	139	145				**1 K 3**			1	82	44			
			7	82	101	3	56	30	1	49	9				3	239	238	**9 K 4**		
1	479	439	8	338	319	4	113	102	2	75	37	3	347	356	5	229	236			
2	404	402				6	100	116	3	564	598	4	260	272	6	67	55	2	271	255
3	235	248	**9 K 0**			7	78	57	4	127	114	5	464	456				3	136	124
4	153	157				8	125	123	5	382	385	7	221	215	**10 K 3**			5	97	126
5	349	354	1	120	102	10	67	63	6	67	55	8	102	80						
6	249	251	2	329	327				7	64	21	10	67	48	2	91	77	**10 K 4**		
7	271	282	3	180	169	**7 K 1**			8	98	82	11	88	100	4	65	38			
8	148	175	4	87	56				9	121	120							1	94	94
			5	158	166	0	123	135	11	215	219	**2 K 3**			**0 K 4**			2	246	238
2 K 0			6	223	237	1	186	200												
			7	111	109	3	186	194	**4 K 2**			2	134	136	6	120	120	**2 K 5**		
0	202	202				4	160	162				3	92	94	8	637	635			
1	369	361	**10 K 0**			5	280	285	0	744	732	4	163	150	10	189	180	8	119	129
2	738	761				6	116	116	1	77	56	6	144	122						
3	112	120	5	60	26	7	208	208	2	514	515	8	156	159	**1 K 4**			**3 K 5**		
4	112	96	6	162	162	9	66	38	3	52	52	10	74	71						
5	419	424							4	547	574				5	252	246	7	149	139
6	455	473	**0 K 1**			**8 K 1**			5	524	531	**3 K 3**			6	174	175	9	240	244
7	55	31							8	237	245				7	204	195			
8	79	79	1	462	449	1	158	131	9	83	76	1	320	319	8	111	131	**4 K 5**		
			3	393	373	2	98	107	10	162	139	3	329	320	9	98	103			
3 K 0			5	223	222	3	159	162				4	140	136	10	88	82	6	88	51
			7	150	152	4	67	24	**5 K 2**			5	129	137				7	178	175
1	514	516	9	66	50	5	78	76				6	91	67	**2 K 4**			8	74	11
2	707	682				6	82	62	1	262	264	7	148	147						
3	185	150	**1 K 1**			7	85	88	2	517	521	8	137	129	5	246	287	**5 K 5**		
5	181	178							3	165	161	9	263	255	6	338	351			
6	290	290	0	569	537	**9 K 1**			4	147	145	10	78	43	8	80	71	6	72	25
7	323	324	1	345	335				5	234	229				9	214	210	7	154	158
8	73	11	3	356	357	1	77	60	6	310	310	**4 K 3**			10	263	252	8	80	4
9	170	162	4	298	287	3	264	259	7	252	253									
			5	467	476	4	85	77	8	148	159	1	160	169	**3 K 4**			**6 K 5**		
4 K 0			7	262	267	5	251	257	9	74	106	2	61	63						
			8	143	140	6	63	28	10	195	184	3	228	226	3	67	91	5	88	26
0	366	371				7	95	93				5	163	165	4	55	16	7	84	36
1	320	321	**2 K 1**						**6 K 2**			6	57	29	5	114	123			
3	105	95				**10 K 1**						7	118	104	6	189	193	**7 K 5**		
4	688	745	1	47	37				0	549	548	8	78	25	7	222	234			
5	211	222	2	158	156	1	83	8	2	53	13	9	57	5	9	101	120	3	131	122
7	64	42	3	47	30	2	115	119	3	78	81				10	75	77	5	176	180
8	283	274	4	192	196	3	61	22	4	366	371	**5 K 3**						6	98	94
9	166	179	5	59	62	4	64	48	8	292	290				**4 K 4**					
			6	164	152	5	66	50	9	78	65	0	275	276				**8 K 5**		
5 K 0			8	187	190	6	75	85				1	378	373	5	142	143			
			10	98	88				**7 K 2**			2	74	70	7	100	32	3	126	117
1	462	476				**11 K 1**						3	189	177	8	216	210	4	66	23
2	160	174	**3 K 1**						1	320	310	4	190	197	9	135	146	5	75	65
3	89	118				1	172	152	2	91	110	5	106	82						
4	58	99	0	387	375	2	80	70	4	57	41	6	75	53	**5 K 4**					
5	164	89	1	442	434	3	119	139	5	100	120	7	182	178						
8	100	84	4	258	253				**8 K 2**						3	74	79			
9	184	183	5	110	105	2	659	700				**6 K 3**			4	81	71			
10	94	55	7	185	183	4	386	375	0	97	62				7	157	166			
			8	138	124	6	419	425	2	351	351	2	173	159	8	103	64			
6 K 0			10	66	83	8	192	181	5	165	156	3	65	41	9	132	141			
			11	152	165	10	380	369	6	278	285	4	98	92						
0	444	438										6	142	120	**6 K 4**					
1	249	260	**4 K 1**			**1 K 2**			**9 K 2**			7	65	59						
2	518	511										8	104	117	0	282	298			
4	277	276	1	342	335	1	402	432	1	336	335				2	384	376			
5	204	209	2	123	122	2	450	441	2	252	203	**7 K 3**			3	118	47			
6	353	346	3	268	263	3	313	308	3	71	66				4	183	180			
8	180	180	5	156	174	5	172	175	6	154	155	0	71	108	5	162	153			
9	212	212	6	62	57	6	142	135				1	157	155	6	274	261			
10	272	277	7	183	195	7	188	199	**10 K 2**			3	161	187	7	69	25			
			8	66	22	9	219	187				4	188	200	8	153	155			
7 K 0						11	147	147	0	214	212	5	278	269						
			5 K 1						1	59	52	6	83	81	**7 K 4**					
1	114	103				**2 K 2**			3	92	91	7	180	177						
3	562	579	0	244	247				4	278	284				2	73	68			
4	107	89	1	475	475	1	451	444				**8 K 3**			3	429	423			
5	307	310	3	156	171	3	211	205	**11 K 2**						4	68	66			
6	112	102	4	256	256	4	929	949				1	153	159	5	239	230			
7	172	176	5	56	55	5	468	474	2	116	97	2	103	122	7	136	148			
8	131	117	7	209	215	6	103	102	3	289	277	3	106	124						
9	122	88	8	88	56	7	56	5				4	61	14						

Table 4b. Calculated and observed structure factors for andalusite at 12 kbar; step-scan data.

H	OBS	CALC	H	OBS	CALC	H	OBS	CALC	H	OBS	CALC	H	OBS	CALC	H	OBS	CALC	H	OBS	CALC
0 K 0			**7 K 0**			**4 K 1**			**1 K 2**			**11 K 2**			**8 K 3**			**7 K 4**		
0	0	3200	5	319	327	7	200	202	1	445	437	2	124	112	1	161	168	3	440	440
2	150	158	6	117	112				3	212	210	3	298	299	2	116	125	5	248	251
4	476	477	7	195	188	**5 K 1**			4	925	909				3	133	135	7	159	162
6	192	188	8	130	119	0	260	254	5	473	472	**0 K 3**			6	76	76			
			9	113	94	1	478	472	6	110	97				7	130	127	**8 K 4**		
1 K 0						3	175	170	8	193	194	5	154	160						
			8 K 0			4	263	269	9	318	322	7	195	215	**9 K 3**			0	440	449
1	440	416				7	228	228				9	105	98				1	103	105
2	394	393	0	574	593	8	86	65	**3 K 2**						0	126	135	4	159	161
3	229	253	1	153	144	9	209	294				**1 K 3**			3	254	246			
4	150	152	4	175	196				3	569	587				5	259	251	**9 K 4**		
5	354	353	7	127	113	**6 K 1**			4	120	110	3	349	361	6	94	62			
6	259	261	8	349	345				5	391	391	4	272	275				1	106	73
7	273	282				2	151	154	8	100	89	5	466	455	**10 K 3**			2	268	275
8	172	180	**9 K 0**			4	123	115	9	119	113	7	221	219				3	134	146
						6	116	110	11	229	235	8	96	83	2	105	90	5	147	139
2 K 0			1	84	99	8	132	131				11	122	117						
			2	336	344				**4 K 2**						**0 K 4**			**10 K 4**		
0	193	198	3	196	189	**7 K 1**						**2 K 3**								
1	351	355	5	170	176				0	741	711				6	120	126	1	96	96
2	709	709	6	239	247	0	151	135	1	89	67	2	136	138	8	653	663	2	252	253
3	113	117	7	123	117	1	195	200	2	503	510	3	106	93	10	200	188			
4	97	98				3	212	208	4	549	573	4	160	153				**2 K 5**		
5	426	423	**10 K 0**			4	165	169	6	539	541	6	131	125	**1 K 4**					
6	465	475				5	285	293	8	253	257	8	167	175				8	145	137
8	89	97	6	190	170	6	131	129	9	128	81	10	97	71	5	263	250			
						7	226	210	10	160	154				6	176	178	**3 K 5**		
3 K 0			**0 K 1**									**3 K 3**			7	198	207			
						8 K 1			**5 K 2**						8	123	134	7	156	155
1	503	502	1	423	429							1	309	323	9	122	109	9	250	259
2	698	652	3	385	372	0	140	135	1	270	268	3	327	320	10	95	93			
3	164	144	5	229	224	2	113	114	2	517	521	4	142	140				**4 K 5**		
5	192	187	7	166	160	3	174	171	3	169	168	5	136	139	**2 K 4**					
6	294	299				7	112	99	4	142	144	6	88	70				6	78	52
7	333	326	**1 K 1**						5	245	242	7	159	149	5	302	300	7	179	184
9	171	168				**9 K 1**			6	323	321	8	141	137	6	345	364			
			0	517	505				7	253	254	9	265	270	9	219	221	**5 K 5**		
4 K 0			1	321	327	0	216	217	8	263	165				10	279	275			
			3	351	352	1	77	61	9	128	114	**4 K 3**						7	173	173
0	362	367	4	294	287	3	263	264	10	209	197				**3 K 4**					
1	320	318	5	272	474	4	91	85				1	166	167				**6 K 5**		
3	108	101	7	276	282	5	268	269	**6 K 2**			3	233	234	3	92	92			
4	696	727	8	147	142	7	104	107				5	153	160	5	140	135	4	79	73
5	216	219							0	550	557	7	125	111	6	207	204			
8	287	295	**2 K 1**			**10 K 1**			4	385	375				7	247	245	**7 K 5**		
9	187	187							8	298	307	**5 K 3**			9	135	129			
			1	51	42	2	120	130							10	117	99	3	151	131
5 K 0			2	145	154	6	112	99	**7 K 2**			0	282	278				4	82	64
			4	197	198							1	375	374	**4 K 4**			5	185	187
1	469	473	5	83	72	**11 K 1**			1	322	323	3	184	180				6	124	109
2	176	177	6	154	155				2	117	107	4	210	207	4	572	564			
3	140	128	8	158	205	1	143	162	5	128	126	5	93	86	5	149	149	**8 K 5**		
4	101	99	10	100	90	3	136	139	7	271	294	7	187	184	8	227	227			
6	111	102										8	105	103	9	168	159	3	133	126
7	243	242	**3 K 1**			**0 K 2**			**8 K 2**			9	234	238						
8	123	86													**5 K 4**					
9	197	204	0	370	369	6	428	429	0	75	60	**6 K 3**								
			3	365	349	8	176	181	6	299	298	6	121	110	5	97	98			
0	438	441	4	254	254	10	377	381				8	107	119	7	170	177			
1	254	263	5	117	112				**9 K 2**						9	151	160			
2	533	511	7	194	193	**1 K 2**						**7 K 3**								
4	296	287	8	134	132				1	342	351				**6 K 4**					
5	204	210	9	252	333	1	390	422	2	225	218	0	124	103						
6	362	363	10	70	83	2	458	435	6	160	161	1	144	152	0	297	303			
8	211	199	11	185	180	3	311	310				3	204	204	2	395	387			
9	221	226				5	178	177	**10 K 2**			4	199	205	4	196	199			
10	291	291	**4 K 1**			6	148	148				5	271	280	5	152	154			
						7	206	204	0	224	222	6	113	85	6	277	275			
7 K 0			1	337	332	9	219	199	3	106	98	7	181	173	8	156	170			
			2	119	118	10	87	90	4	306	302									
1	102	101	3	274	269	11	145	152												
3	576	579	5	178	174															

Table 4c. Calculated and observed structure factors for andalusite at 25 kbar; step-scan data.

H	OBS	CALC	H	OBS	CALC	H	OBS	CALC	H	OBS	CALC	H	OBS	CALC	H	OBS	CALC	H	OBS	CALC
	H 0 0			H 6 0			H 5 1			H 3 2			H 0 3			H 8 3			H 8 4	
2	216	195	2	423	469	0	232	227	1	316	313	5	263	269	2	179	158	0	629	536
4	396	372	3	269	275	1	468	467	2	213	208	7	114	108	3	120	130	1	103	116
6	442	438	4	81	62	3	110	107	3	566	589	9	141	124	5	143	95	4	241	218
8	547	561	5	104	109	4	171	167	5	161	164				6	122	102	6	149	152
0	108	57	6	363	344	7	282	287	6	109	80		H 1 3							
			7	95	109	8	110	84	10	121	96					H 9 3			H 9 4	
	H 1 0		8	241*	12	9	249	246				3	231	313						
			9	210	217					H 4 2		4	174	166	2	107	14	1	118	101
1	476	420	10	177	159		H 6 1					5	365	372	3	259	253	2	205	199
2	370	356							0	368	367	7	143	144	5	211	218	3	142	123
3	530	502		H 7 0		2	156	150	2	884	913	8	129	151				4	145	141
4	337	319				6	120	113	4	547	564					H 11 3		5	133	152
5	468	461	5	225	226	7	126	123	5	115	127		H 2 3							
6	254	253	7	175	183	9	106	19	6	364	369				1	119	103		H 10 4	
7	87	105	8	114	103	10	110	86	10	287	274	1	78	45						
8	147	132	9	103	92							2	142	141		H 2 4		0	176	167
0	126	109					H 7 1			H 5 2		5	87	70				1	142	103
1	123	114		H 0 1		0	163	164	1	157	167	6	162	158	5	119	127	2	249	232
						1	250	264	2	445	458	8	122	114	6	380	382			
	H 2 0		1	512	511	3	189	194	3	373	383				7	100	80		H 5 5	
			3	371	372	4	187	184	5	216	231		H 3 3		9	245	261			
0	170	159	5	250	247	5	217	209	8	155	159				10	150	238	2	109	88
1	399	402	7	155	132	7	215	203	10	133	102	0	307	297				5	111	20
2	720	716	9	196	201	9	138	99				1	348	355		H 3 4		7	159	177
3	714	655								H 6 2		2	91	83						
5	165	171		H 1 1			H 8 1					3	309	312	2	110	95		H 6 5	
6	490	498							0	410	415	4	213	227	3	94	102			
7	110	107	0	467	431	1	158	144	1	149	146	5	176	175	5	93	97	2	139	109
9	325	329	1	335	329	2	187	187	2	105	94	7	196	196	6	98	57	4	104	44
0	287	296	2	74	45	3	121	127	4	502	519	8	115	118	7	414	432	7	111	103
1	108	86	3	422	428	6	100	107	5	304	307	9	239	233	9	132	136			
			4	330	326				7	126	64								H 7 5	
	H 3 0		5	439	471		H 9 1		8	282	277		H 4 3			H 4 4				
			7	180	189				9	159	145							1	196	192
1	222	246	8	126	125	3	295	318				1	270	267	0	180	175	3	148	147
2	112	116	11	159	159	5	261	271		H 7 2		2	185	147	4	554	549	4	176	173
3	202*	137										3	137	139	5	105	71	5	166	153
4	103	93		H 2 1			H 0 2		1	200	201	5	194	198	6	198	195			
5	128	121							5	241	239	6	95	79	7	101	66		H 8 5	
6	86	77	1	110*	32	4	653	709	7	273	274	7	190	199	8	156	154			
7	568	564	2	153	154	6	556	542	9	235	240	9	107	68				2	129	130
9	182	174	3	85	75	10	207	215								H 5 4				
			4	116	115					H 8 2			H 5 3		1	263	262		H 9 5	
	H 4 0		6	149	146		H 1 2								2	286	287			
			8	122	103				0	162	172	0	143	160	3	145	139	3	256	238
0	497	470	10	129	121	1	411	430	1	163*	39	1	437	447	4	142	151			
1	143	147				2	432	438	2	194	191	3	137	142	6	149	139			
2	93	96		H 3 1		5	267	266	4	239	245	4	154	156	7	221	239			
4	702	716				7	322	311	5	155	165	7	270	272	9	186	124			
5	93	98	0	383	364	9	329	327	6	292	277	9	219	229						
6	260	268	1	353	357				8	123	22					H 6 4				
7	97	88	3	344	341		H 2 2						H 6 3		0	138	128			
8	175	188	4	258	264					H 9 2		2	151	124	1	174	174			
			5	167	171	0	676	683				3	111	72	2	366	363			
	H 5 0		7	202	206	1	455	432	1	182	192	6	122	115						
			8	148	154	8	347	348				0	295	220		H 7 4				
1	221	224		H 4 1		9	204	200		H 10 2		1	213	211						
3	187*	73				11	135	111				3	139	154	1	207	200			
4	192	188	1	295	289				0	362	360	4	113	98	3	244	243			
5	178	155	2	191	191				2	124	38	5	169	172	5	171	175			
			3	258	233				4	170	144	7	164	169	7	132	149			
			5	252	259				5	168	172	8	140	120						
			7	171	162															

Table 4d. Calculated and observed structure factors for andalusite at 37 kbar; continuous scan data.

H	OBS	CALC
H 0 0		
0	0	3200
2	204	201
4	377	373
6	448	441
8	597	595
10	69	49
H 1 0		
1	453	434
2	375	366
3	534	510
4	311	318
5	508	479
6	272	253
7	96	107
8	137	137
9	103	107
10	94	102
11	121	119
H 2 0		
0	155	147
1	407	410
2	733	734
3	720	683
4	48	17
5	173	173
6	517	518
7	113	103
9	304	335
10	324	318
11	105	75
H 3 0		
1	225	244
2	76	106
3	187	149
4	104	104
5	121	124
6	89	81
7	587	581
8	84	82
9	192	181
10	79	34
H 4 0		
0	496	482
4	751	742
6	262	274
7	74	94
8	197	202
H 5 0		
3	189	188
4	205	226
5	93	77
6	222	216
7	324	325
9	159	152
H 6 0		
0	168	172
1	254	252
2	508	505
3	283	285
4	56	60
5	116	106
6	355	359
7	86	89
8	74	24
9	244	241
H 6 0		
10	170	178
H 7 0		
1	282	290
2	71	48
3	326	321
5	224	228
7	183	176
8	96	115
9	99	111
H 8 0		
4	289	300
5	115	116
6	189	193
7	110	114
H 0 1		
1	504	527
3	384	381
5	259	269
6	63	0
7	144	148
9	208	212
11	96	92
H 1 1		
0	440	439
1	316	327
2	44	45
3	418	434
4	373	328
5	495	488
7	189	194
8	116	125
11	172	173
H 2 1		
1	40	41
2	138	148
4	93	116
6	142	139
8	116	121
10	126	125
11	83	65
H 3 1		
0	389	371
1	360	363
3	372	344
4	278	262
9	266	263
11	137	147
H 4 1		
1	370	291
3	259	265
5	278	266
6	110	105
7	133	159
9	103	92
H 5 1		
0	242	225
1	480	481
3	106	113
4	167	157
5	87	55
7	303	307
8	84	84
9	272	264
H 6 1		
2	133	142
4	68	62
6	108	108
7	120	117
8	82	79
H 7 1		
0	185	166
1	275	270
2	80	18
3	192	200
4	162	170
5	222	229
6	77	46
7	205	217
8	92	92
9	113	110
H 8 1		
1	124	144
2	193	202
3	129	127
6	120	127
7	82	56
H 9 1		
3	314	330
5	294	289
H 10 1		
2	87	81
3	101	74
4	72	27
6	98	60
H 0 2		
4	769	735
6	569	560
8	76	66
10	247	230
H 1 2		
1	413	439
2	440	452
4	71	56
9	360	345
10	71	57
H 2 2		
0	683	711
1	451	433
4	512	529
5	523	519
7	124	103
8	355	364
9	217	219
11	102	103
H 3 2		
1	329	317
3	599	601
4	87	62
5	168	169
6	84	77
9	79	66
10	96	92
H 4 2		
0	384	374
2	931	950
3	101	84
4	565	579
5	129	133
6	387	383
7	79	65
10	295	296
H 5 2		
1	182	167
2	470	478
3	393	400
5	227	226
7	120	135
8	150	151
10	118	122
H 6 2		
0	419	415
1	185	163
3	73	78
4	497	523
5	312	336
7	81	68
8	295	300
9	154	149
H 7 2		
1	205	199
5	255	261
7	287	279
9	251	253
H 8 2		
0	166	161
2	196	192
3	96	89
4	263	257
5	166	162
6	305	291
7	74	77
H 9 2		
1	209	206
5	109	121
7	78	73
H 10 2		
0	402	385
1	88	76
2	78	43
3	85	74
4	172	171
5	178	182
H 11 2		
1	151	159
3	228	223
H 0 3		
5	297	289
7	112	116
9	141	135
H 1 3		
3	312	324
4	159	166
5	401	385
6	68	5
7	141	142
8	155	163
9	70	47
H 2 3		
2	135	145
4	79	67
6	150	151
8	140	130
10	97	90
H 3 3		
0	280	290
1	357	366
2	87	74
3	284	315
4	246	238
5	172	175
7	206	207
8	115	120
9	234	246
10	78	59
H 4 3		
1	266	282
2	165	164
3	162	155
4	61	32
5	178	201
6	96	90
7	200	198
9	94	88
H 5 3		
0	170	166
1	468	452
3	140	130
4	130	145
6	74	68
7	291	289
8	74	62
9	243	249
2	133	121
6	130	114
7	78	74
8	88	91
H 7 3		
0	207	220
1	190	201
3	158	149
4	90	90
5	183	178
7	180	175
8	121	122
H 8 3		
1	88	86
2	160	178
H 8 3		
3	138	136
5	118	106
6	132	113
H 9 3		
0	82	78
3	271	270
5	245	239
H 10 3		
3	76	41
H 11 3		
0	82	53
1	122	124
H 1 4		
7	68	77
8	86	104
9	91	83
10	92	88
H 2 4		
5	150	143
6	405	390
7	77	79
9	270	270
10	241	252
H 3 4		
3	93	97
4	56	56
5	95	91
6	66	64
7	443	448
9	138	143
H 4 4		
1	66	93
2	76	55
4	589	571
5	91	73
6	185	196
7	72	73
8	164	157
H 5 4		
1	270	265
2	304	300
7	251	255
H 6 4		
0	104	129
1	182	180
2	392	380
3	191	188
5	95	88
6	276	273
7	85	69
H 7 4		
1	198	210
3	256	248
5	177	178
7	161	152
H 8 4		
0	652	666
1	96	110
2	93	78
4	234	229
5	92	93
6	163	156
H 9 4		
1	96	107
2	222	224
3	132	124
4	154	157
5	168	164
H 10 4		
0	164	166
1	107	117
2	258	255
H 4 5		
7	94	67
H 5 5		
4	97	86
6	72	41
7	195	195
H 6 5		
2	118	110
4	78	52
5	81	26
7	115	109
H 7 5		
0	68	61
1	190	198
2	83	21
3	167	156
4	163	170
5	167	173
H 8 5		
1	136	133
2	135	142
3	70	71
H 9 5		
3	253	251